

Power in a Balanced Three-Phase System

- ❑ To find total power in a balanced system
 - Determine power in one phase
 - Multiply by three

- You can also use single-phase equivalent in power calculations
 - Power will be power for just one phase



Three-Phase Active (Average) Power

□ Active power per phase = $V_w I_w$ x power factor

□ Total active power = $3V_w I_w$ x power factor

$$P = 3V_w I_w \cos \theta$$

➤ If I_L and V_L are rms values for line current and line voltage respectively. Then for delta (Δ) connection: V_ϕ

= V_L and $I_w = I_L / \sqrt{3}$. therefore: $P = \sqrt{3} V_L I_L \cos \theta$

➤ For star connection (Y) : $V_w = V_L / \sqrt{3}$ and $I_\phi = I_L$. therefore:

$$P = \sqrt{3} V_L I_L \cos \theta$$



Three-Phase Instantaneous Power

Instantaneous Phase Voltages

$$v_{an}(t) = V_m \sin(\check{S} t)$$

$$v_{bn}(t) = V_m \sin(\check{S} t - 120^\circ)$$

$$v_{cn}(t) = V_m \sin(\check{S} t - 240^\circ)$$

Instantaneous Phase Currents

$$i_a(t) = I_m \sin(\check{S} t - \theta)$$

$$i_b(t) = I_m \sin(\check{S} t - \theta - 120^\circ)$$

$$i_c(t) = I_m \sin(\check{S} t - \theta - 240^\circ)$$

$$v_{an}(t) = \sqrt{2}V \sin \check{S} t$$

$$v_{bn}(t) = \sqrt{2}V \sin(\check{S} t - 120^\circ)$$

$$v_{cn}(t) = \sqrt{2}V \sin(\check{S} t - 240^\circ)$$

$$i_a(t) = \sqrt{2}I \sin(\check{S} t - \theta)$$

$$i_b(t) = \sqrt{2}I \sin(\check{S} t - 120^\circ - \theta)$$

$$i_c(t) = \sqrt{2}I \sin(\check{S} t - 240^\circ - \theta)$$



Three-Phase Instantaneous Power

✓ Instantaneous Power

$$p(t) = v(t)i(t)$$

➤ Therefore, the instantaneous power supplied to each phase is:

$$p_a(t) = v_{an}(t)i_a(t) = 2VI \sin(\check{S}t) \sin(\check{S}t - \theta)$$

$$p_b(t) = v_{bn}(t)i_b(t) = 2VI \sin(\check{S}t - 120^\circ) \sin(\check{S}t - 120^\circ - \theta)$$

$$p_c(t) = v_{cn}(t)i_c(t) = 2VI \sin(\check{S}t - 240^\circ) \sin(\check{S}t - 240^\circ - \theta)$$

▪ Since

$$\sin r \sin s = \frac{1}{2} [\cos(r - s) - \cos(r + s)]$$



Three-Phase Instantaneous Power

▪ Therefore

$$p_a(t) = VI [\cos \theta - \cos(2\check{S}t - \theta)]$$

$$p_b(t) = VI [\cos \theta - \cos(2\check{S}t - 240^\circ - \theta)]$$

$$p_c(t) = VI [\cos \theta - \cos(2\check{S}t - 480^\circ - \theta)]$$

➤ The total instantaneous power

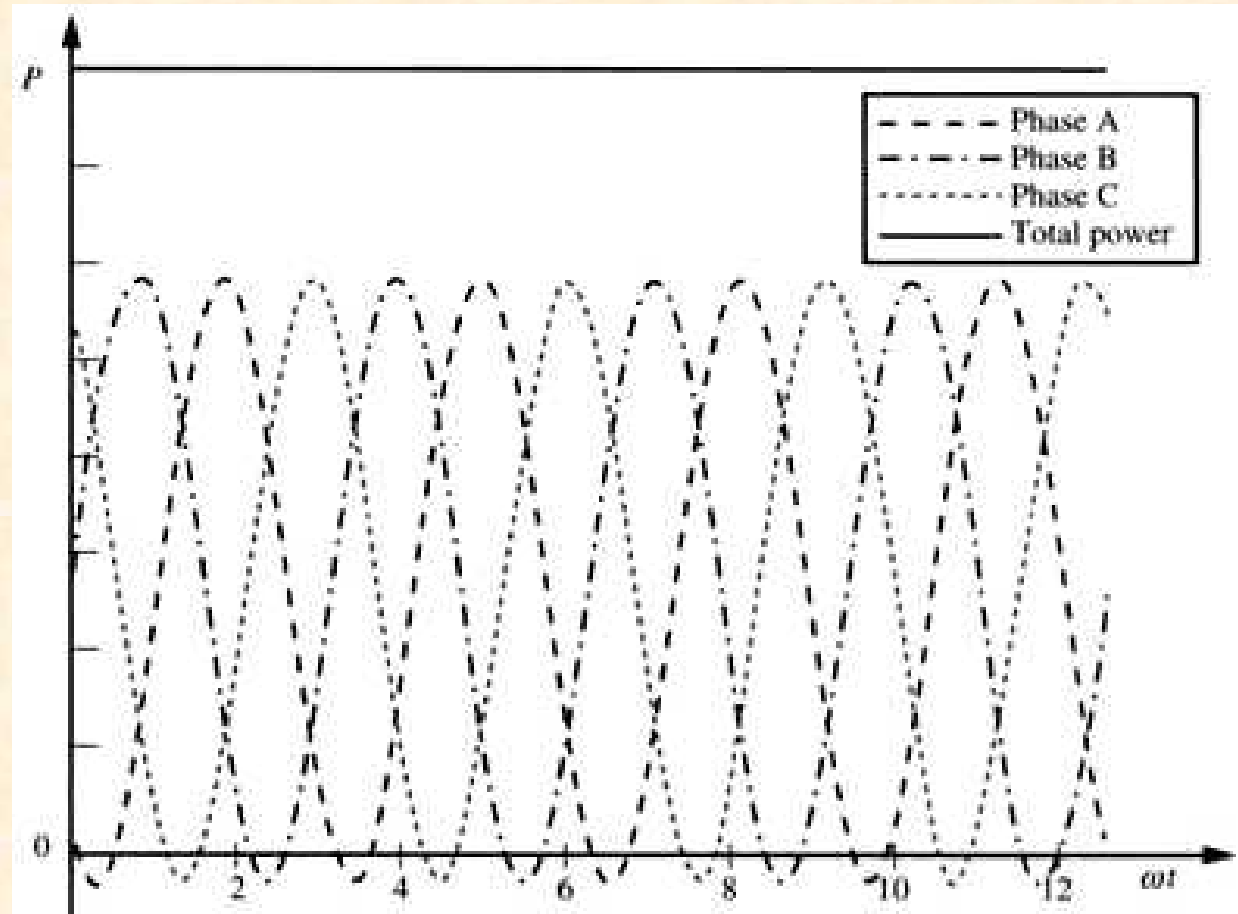
$$p_{tot}(t) = p_a(t) + p_b(t) + p_c(t) = 3VI \cos \theta$$

Note that: the pulsing components cancel each other because of 120° phase shifts.

✓ For a balanced three phase circuit the instantaneous power is constant



Three-Phase Instantaneous Power



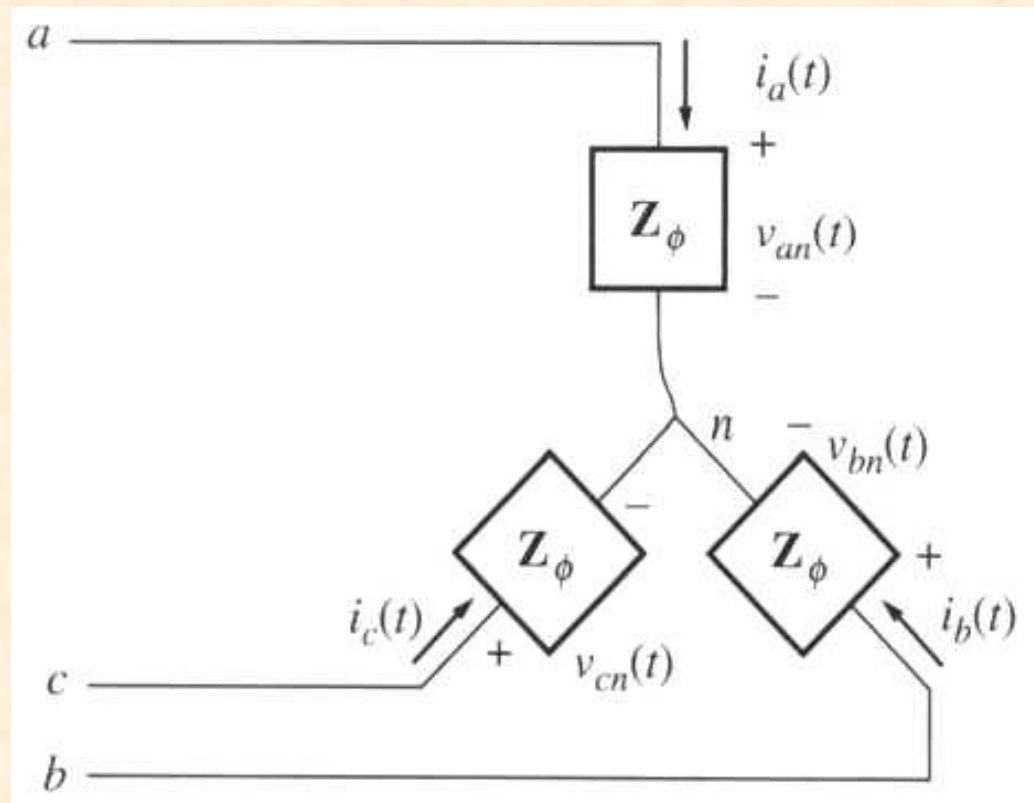
✓ power in phases
is **Time Variant**

✓ The total power supplied
to the load is **constant**



Power Relationships For a balanced Three-Phase load

- For a balanced Y-connected load with the impedance $Z_w = Z \hat{e}_{''}^0$:



Power Relationships For a balanced Three-Phase load

□ Using **Phase** quantities in each phase of a Y- or U-connection

✓ Real Power:

$$P = 3V_W I_W \cos \theta = 3I_W^2 Z \cos \theta \quad 3I_W^2 R$$

✓ Reactive Power:

$$Q = 3V_W I_W \sin \theta = 3I_W^2 Z \sin \theta \quad 3I_W^2 X$$

✓ Apparent Power:

$$S = 3V_W I_W = 3I_W^2 Z$$



Power Relationships For a Balanced Three-Phase load

□ Using **Line** quantities of a Y-connected Load

✓ Real Power

$$P = 3V_W I_W \cos \theta$$

➤ Since for this load $I_L = I_W$ and $V_W = V_L / \sqrt{3}$

➤ Therefore: $P = 3 \frac{V_L}{\sqrt{3}} I_L \cos \theta$

➤ Finally: $P = \sqrt{3} V_L I_L \cos \theta$



Power Relationships For a Balanced Three-Phase load

□ Using **Line** quantities of a U-connected Load

✓ Real Power

$$P = 3V_W I_W \cos \theta$$

➤ Since for this load $V_L = V_W$ and $I_W = I_L / \sqrt{3}$

➤ Therefore: $P = 3 V_L \frac{I_L}{\sqrt{3}} \cos \theta$

➤ Finally: $P = \sqrt{3} V_L I_L \cos \theta$

The same as for a
Y-connected load!



Power Relationships For a Balanced Three-Phase load

□ Using **Line** quantities of Y- or U-connection

➤ Reactive power: $Q = \sqrt{3}V_L I_L \sin \theta$

➤ Apparent power: $S = \sqrt{3}V_L I_L$

□ Note: θ is the angle between the phase voltage and the phase current (the impedance angle).

✓ Power factor is: $F_p = \cos \theta = P/S = P_w/S_w$

